1. Dynamics

Vectors and Scalars

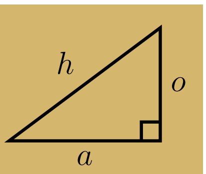
Scalar quantities are defined by a magnitude (size)—usual rules for addition, subtraction etc. of real numbers apply. **Vector** quantities have both magnitude and direction, and may be one, two or three dimensional. Use trigonometry or a scale diagram to add vectors in two dimensions.

Trigonometry of right-angled triangles

$$a^2 + o^2 = h^2 \quad \text{(Pythagoras)}$$

$$\sin\theta = \frac{o}{h} \quad \cos\theta = \frac{a}{h} \quad \text{(soh-cah-toa)}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{o}{a}$$



N.B. Make sure your calculator is set to degrees.

Exercise 1.1. Sort the following quantities and their units *velocity, distance, temperature, force, speed, weight, acceleration, mass, energy, displacement, time* and *m, ms^{-1}, ms^{-2}, kg, N, J, s, °C*

Scalar	Vector

Example 1.2. A hiker walks 3.00 km North and then 4.00 km East. Their displacement from the starting point is $\sqrt{3^2+4^2} = 5.00$ km at an angle of $\theta = \tan^{-1}(4.00/3.00) = 53.1°$ from due North (alternatively, draw 3 cm up and 4 cm across with a ruler, and measure 5 cm $\to 5.00$ km and $53°$ using a protractor). The *distance* travelled by the hiker is $3.00 + 4.00 = 7.00$ km.

Speed-distance-time and acceleration

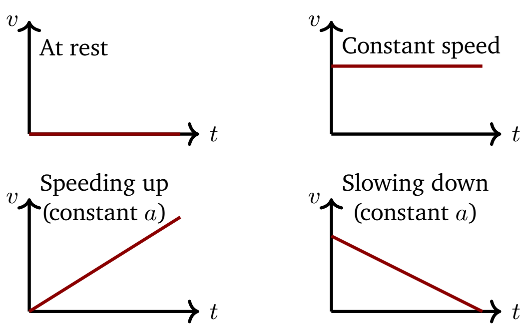
Average **velocity** is displacement divided by time. Average **speed** is the magnitude of displacement i.e. distance divided by time. Average **acceleration** is the change in velocity (final v minus initial u) divided by time.

$$v = \frac{s}{t} \quad |v| = \frac{d}{t} \quad a = \frac{v-u}{t}$$

Instantaneous velocity (speed) or acceleration can be approximated by measuring average velocity (speed) or acceleration over smaller and smaller time intervals (see below).

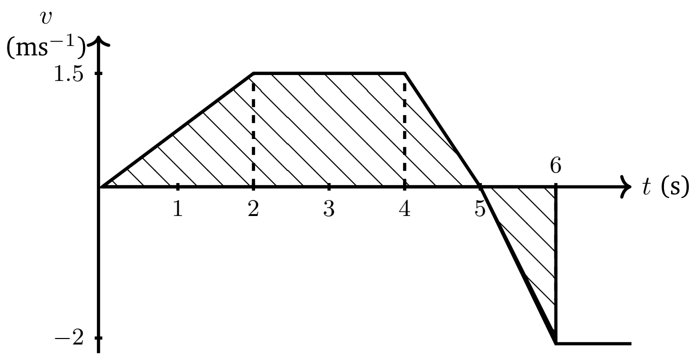
Speed-time graphs (1D)
Provide information on the motion of the object. The gradient (slope) of lines is the acceleration and the signed area between lines and the t-axis the net displacement.

Horizontal lines indicate motion at constant speed (or velocity) i.e. zero acceleration, upwards and downwards straight lines indicate increasing and decreasing speed (or velocity) with constant acceleration



Exercise 1.3. A runner completes one lap of a 400 m athletics track in 52 seconds. What was their (a) average speed and (b) average velocity?

Example 1.4. The velocity of a slot car along a track is plotted below. How far is the car from its starting position at $t = 6$? How far does it travel over the interval $t = 2$ to $t = 4$?



The displacement is the sum of areas

$$\frac{1}{2} \times 2 \times 1.5 + 2 \times 1.5 - \frac{1}{2} \times 1 \times 2 = 3.5\text{m}$$

Exercise 1.5. Determine the maximum speed of the car and its acceleration in the first two seconds.

Measuring speed and acceleration

A light gate consists of a light source and a photocell and may be connected to an electronic timing device or computer. The timing device is triggered by the light beam falling the photocell or being blocked by a card.

By recording the time for a card attached to an object blocks the beam, the speed of that object can be obtained as

$$\text{instantaneous speed} = \frac{\text{card length}}{\text{time beam is blocked}}$$

Exercise 1.6. The times at which a card of length 5 cm attached to a cart blocks the light beam are $t = 0.8$, $t = 0.95$ s. Determine the velocity of the cart in ms^{-1}.

Example 1.7. A double card (or card with a cut-out) can be used to measure the instantaneous velocity of an object twice in succession and hence its acceleration $a = (v-u)/\Delta t$ where Δt is the time *between* velocity readings.

Newton's laws

(N1) A body remains at rest or in uniform motion (straight line, constant speed) unless acted upon by a net force.

(N2) The acceleration a of a body of mass m subject to a net force F_{Net} is $F_{\text{Net}} = ma$.

(N3) If two bodies exert forces on each other, these forces have equal magnitude and opposite direction.

N.B. In (N1) and (N2) F_{Net} is the net or *resultant* force, determined by summing all (vector) forces acting on a body. In situations where (N1) holds i.e. $F_{\text{Net}} = 0$, the body is in equilibrium and force-balance can often be used to determine unknown forces.

Weight

A common force is the **weight** of a body, which is the gravitational force acting on that body in a gravitational field e.g. on Earth or another planet,

$$W_g = mg$$

with g the gravitational field strength (9.8 ms^{-2} on Earth).

In rocket flight one must account for the fact that gravitational field strength decreases with height (as does air resistance), so a rocket's acceleration with generally increase.

Exercise 1.8. Calculate the net force acting on a F1 driver of mass $m = 72$ kg that reaches a speed of 100 km hr^{-1} in 2.5 s (hint: convert speed to ms^{-1} and calculate a). Are there any vertical forces acting on the driver?

Example 1.9. Whilst standing on an ice rink, Anne ($m_A = 60$ kg) pushes Bill ($m_B = 70$ kg) to the right with a force 140 N for 0.5 s. Bill beings to travel to the right with an acceleration of $a_B = 140/m_B = 2$ ms^{-2} (N2). By (N3) Anne experiences a force of 140 N to the left, hence $a_A = -140/m_A = -2.33$ ms^{-1} (N2). Assuming no friction, after the 0.5 s has passed, Bill and Anne continue to travel to the right and left (respectively) at constant speed (N1).

Exercise 1.10. A skydiver of mass $m = 85$ kg reaches a terminal velocity of 50 ms^{-1}. What is the size of the upwards drag force (air resistance) acting on the skydiver ($g = 9.8$ ms^{-2})?

Example 1.11. The gravitational acceleration on the moon is about $g_{\text{moon}} = 1.6$ ms^{-2}, so you weigh about $g/g_{\text{moon}} = 9.8/1.6 \approx 6$ times less on the moon than on Earth.

Work and energy

The **work** done W by a *constant* force F acting on a body is $W = Fs$ where s is displacement *in the direction of the force.*

Kinetic energy E_k is the energy a body posses due to its motion, equal to the work the body can do in coming to rest. **Gravitational potential energy** E_p is the energy of a body due to its position (height h) in a gravitational field.

$$W = Fs \quad E_k = \frac{1}{2}mv^2 \quad E_p = mgh$$

Exercise 1.12. Calculate the total work done by delivery driver pushing a box 1 m East with a horizontal force of 200 N and then 3 m North with a force of 100 N. Does the gravitational force do any work on the box during this motion?

Example 1.13. A weightlifter applying a vertical force of 1200 N to lift a weight 0.5 m does $1200 \times 0.5 = 600$ J of work against the gravitational force.

Work and energy (cont.)

Work-energy theorem
The work done by the net force acting on a body is equal to the change in kinetic and potential energies of that body,

$$W_{\text{Net}} = \Delta E_k + \Delta E_p = \frac{1}{2}m(v^2 - u^2) + \frac{1}{2}mg(h_2 - h_1)$$

If no external forces act, the sum of kinetic and potential energies is constant (conservation of energy).

Exercise 1.14. What is the mass of the weight in the previous example?

Example 1.15. A sledge of mass $m = 15$ kg sliding along a rough horizontal surface is brought to rest from $u = 2$ ms^{-1} by a constant frictional force f over a distance of 3 m. The change in kinetic energy is $(1/2)(0^2 - 2^2) = -2$J equals the work done by f (work-energy theorem), hence $f = -2/3$ N i.e. $2/3$ N in the direction opposing the sledge's motion.

Projectile motion

Projectile motion can be resolved into two components: horizontal motion with a constant velocity ($F_h = 0 \rightarrow a_h$, N2) and vertical motion with a constant downward acceleration $a_h = g$. This is valid under the assumption of no air resistance (reasonable for small particle-like masses).

$$\begin{aligned} s_h &= v_h t && \text{(horizontal distance)} \\ v_v &= u_v + gt && \text{(downward velocity)} \end{aligned}$$

The area under a v_h-t graph gives the horizontal range and that under a v_v-t graph the vertical height (lost).

Exercise 1.16. Calculate the distance travelled by a ball thrown horizontally with a speed 2 ms^{-1} from a window if the time of flight is 1.5 s. What is the vertical speed of the ball upon hitting the ground ($g = 9.8$ ms^{-2})?

Example 1.17. A satellite is a project which is falling towards Earth at the same rate the Earth's surface curves, so it never gets any closer and is said to be in orbit (*geostationary* if its orbital speed matches that of the Earth's rotation).