

4. Properties of matter

Specific heat capacity

The heat energy required to raise the temperature of a body is determined by the mass m of the body and the type of material:

$$E_h = mc\Delta T$$

where c is the **specific heat capacity**, which is the energy required to raise the temperature of a unit mass of a substance by one degree Celsius (units $\text{Jg}^{-1}\text{C}^{-1}$ or $\text{Jkg}^{-1}\text{C}^{-1}$).

Conservation of energy may be applied to determine the temperature change of bodies 1, 2, ... exchanging heat, given their masses and specific heat capacities:

$$\Delta E_h = m_1c_1\Delta T_1 + m_2c_2\Delta T_2 + \dots = 0$$

In particular, if the bodies are allowed to reach thermal equilibrium then their final temperatures will be the same.

Specific latent heat

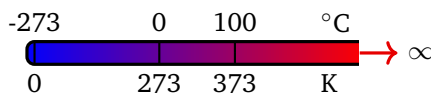
In addition to the heat required to produce temperature rise in a stable material we can also consider the heat required to change the *state* of a material, from solid to liquid (fusion or melting) or from liquid to gas (vaporisation or boiling).

This is captured by the **specific latent heat** of the material l_f or l_v , such that

$$E_f = ml_f \quad E_v = ml_v$$

In words: l_f (or l_v) is the energy required to change a unit mass of a substance from solid to liquid (or liquid to gas) *without* any temperature change. The last point is important; melting and boiling take place at a constant temperature ($\Delta T = 0$).

Temperature

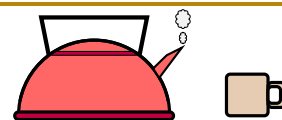


Temperature T is a measure of the *average kinetic energy* of the particles comprising a system. Informally, we associate it with the 'hotness' or 'coldness' of an object.

Temperature of bulk matter may be measured using a **thermometer**. The most common temperature scales are Fahrenheit ($^{\circ}\text{F}$), Celsius ($^{\circ}\text{C}$) and Kelvin (K). These are related by

$$T(\text{F}) = \frac{5}{9}(T(\text{C}) - 32), \quad T(\text{C}) = T(\text{K}) - 273.15$$

Kelvin is an *absolute* scale, 0 K (-273.15°C) being the lowest theoretically possible temperature, and is most often used for scientific purposes.



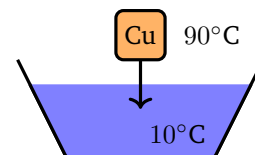
Exercise 4.1. Calculate the energy required to bring one cup 400 ml of water from 20°C^{-1} to 100°C^{-1} (the density of water is 1 g per ml and its $c_W = 4.2 \text{ Jg}^{-1}\text{C}^{-1}$). How long would this take for a 3 kW kettle?

Example 4.2. A 1 kg block of Copper ($c_{\text{Cu}} = 390 \text{ Jkg}^{-1}\text{C}^{-1}$) at 90°C is dropped into 2 litres of cold water at 10°C ($c_W = 4,200 \text{ Jkg}^{-1}\text{C}^{-1}$). We can calculate the final temperature T_f of the copper and water system from

$$\begin{aligned} \Delta E_h &= m_{\text{Cu}}c_{\text{Cu}}\Delta T_{\text{Cu}} + m_Wc_W\Delta T_W = 0 \\ \Rightarrow 1 \cdot 390 \cdot (T_f - 90) + 2 \cdot 4200 \cdot (T_f - 10) &= 0 \end{aligned}$$

Rearranging for T_f ,

$$\begin{aligned} \Rightarrow (390 + 8400)T_f &= 390 \cdot 90 + 2 \cdot 4200 \cdot 10 \\ \Rightarrow T_f &\approx 13.5^{\circ}\text{C} \end{aligned}$$



Exercise 4.3. Calculate the heat required to melt 2 kg of ice at 0°C given $l_f = 3.3 \times 10^5 \text{ Jkg}^{-1}$ for water. How does this compare to the energy change when the same volume of water later freezes?

Example 4.4. If both a temperature change and a phase change occurs, combine both types of heat calculation. For example, to melt the 2 kg of ice if it started at -10°C we would need, in *addition* to the energy calculated in 4.3,

$$E_h = 2c_{\text{ice}}(0 - -10) = 40 \text{ kJ}$$

where we used the specific heat of ice $c_{\text{ice}} \sim 2 \text{ kJkg}^{-1}$.

Exercise 4.5. Calculate the total energy required to entirely boil (vaporise) the cup of 20°C^{-1} water in Exercise 4.1 ($l_v = 22.6^5 \text{ Jkg}^{-1}$ for water).

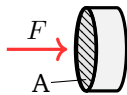
Example 4.6. One can show that, for an ideal monatomic gas,

$$\frac{1}{2}mv_{\text{rms}}^2 = \frac{3}{2}k_B T$$

where v_{rms} is the root mean square speed of the particles of mass m , a measure of their average speed in the gas, and k_B is the Boltzmann constant. Hence we see that the average kinetic energy of the particles in the gas increases with T .

N.B. A 1°C temperature change is the same as a 1°C , but the same *cannot* be said for Fahrenheit. When using the gas laws (next section) it is essential to always convert to Kelvin first.

The kinetic model



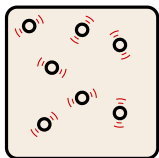
Pressure

In general **pressure** P is defined as force per unit area,

$$P = \frac{F}{A}$$

and is measured in Pascals (Pa), equivalent to Newtons per square meter (Nm^{-2}).

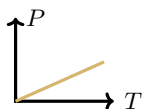
In a gas, pressure arises due to particles of the gas colliding with a boundary e.g. container walls.



Kinetic theory

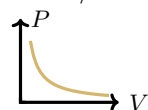
The **kinetic model** describes a gas as a large number of sub-atomic particles in constant, random motion. The particles undergo frequency elastic collisions with each other as well as the walls of any container (generating pressure). If the temperature of the gas increases, the average kinetic energy of the particles increases, leading to more frequent and energetic collisions with the container, hence higher pressure. This explains an observation that P is proportional to *absolute* temperature T ($P \propto T$) or

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \quad (\text{pressure law})$$



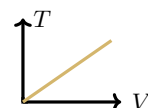
On the other hand, if the volume of the container *decreases* then so will the time between collisions, hence $P \propto 1/V$ or

$$P_1 V_1 = P_2 V_2 \quad (\text{Boyles' law})$$



Finally, if one heats a gas at a fixed pressure (flexible container), the increased energy and frequency of collisions will cause the gas to expand, giving $V \propto T$ or

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad (\text{Charles' law})$$



The above laws can be combined into the general equation

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \quad (\text{combined law})$$

known as the combined gas law, with gases that follow this relationship well referred to as ideal gases. This is a good model for many gases at room (or higher) temperatures provided their density or pressure is not too high.

N.B. In each of the above laws T is the absolute temperature i.e. must be in Kelvin.

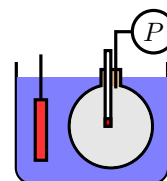
Exercise 4.7. Calculate the pressure on the end of a circular hydraulic press lifting a car with a force of 1.5 kN if its diameter is 0.2 m.

Example 4.8. The pressure under objects can be reduced by increasing the surface area of contact points, for example with wider tyres on an off-road vehicle to avoid sinking into soft terrain.

Exercise 4.9. An 8 L scuba diving tank is filled with N_2 to a pressure of 250 bar (1 bar = 100 kPa) at 25°C. If a diver is submersed in water at 5°C, what would the pressure gauge on the tank read? [hint: remember to convert T to Kelvin!]

Example 4.10. The relationship between P and T can be investigated by submerging an airtight flask contained a fixed mass of gas in a water bath whose temperature may be increased using an immersion heater.

The temperature of the gas inside the flask may be monitored with a thermometer and the pressure with a Bourdon pressure gauge. Increasing the temperature of the gas slowly via the heater, P can be recorded for many different T .



As the volume of gas is a constant, plotting P against T should passing a straight line (passing through O is absolute temperature units are used).

A similar experiment can be used to demonstrate $V \propto T$ by replacing the flask with a glass tube of gas trapped by a small quantity of mercury (allowing the gas to expand at a constant P ; a ruler can be used to measure changes in volume).

Exercise 4.11. Describe an experiment that could be used to demonstrate Boyles' law, including a diagram of the apparatus used and a sketch of P against V .

Example 4.12. A weather balloon contains a fixed amount of gas and has a volume of 3 L at atmospheric pressure when $T = 5^\circ\text{C}$. What will be volume of the same balloon on a hot day (30°C)? Assuming P is constant, using Charles' law

$$V_2 = T_2 \frac{V_1}{T_1} = (30 + 273) \frac{3}{5 + 273} = 3.3 \text{ L}$$

Exercise 4.13. At the start of the compression stroke an engine piston contains 380 cm^3 of air and vaporised gasoline at 1 bar and a temperature of 27°C . At the end of the stroke, the gas has been compressed to 40 cm^3 and the pressure has increased to 2.7 bar. Determine the final temperature.