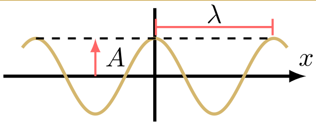
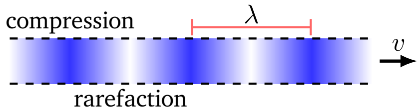
5. Waves & radiation

Waves





Waves are propagating disturbances in one or more quantities. **Mechanical waves** require a physical medium (matter) to propagate whereas **electromagnetic waves** do *not*.

Mechanical waves may be **longitudinal**, meaning particles in the medium are displaced *parallel* to the direction of propagation as the wave passes, or **transverse**, with particle displacements *perpendicular* to propagation. In both cases there may be transfer of energy and momentum in the direction of propagation *without* net movement of particles.

Electromagnetic (EM) waves are *transverse* oscillations of electric and magnetic fields. These exist independently of a physical medium, hence the capability of EM waves to transfer energy and momentum through even a vacuum.

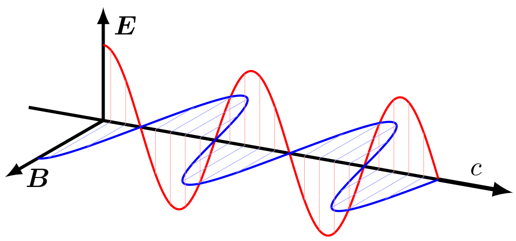
Periodic waves

In *periodic* waves the wave motion repeats after a certain *time*—the **period** T—and distance—the **wavelength** λ. These are related to the *speed* v at which the wave travels via $v = \lambda/T$ or

$$\boxed{v = f\lambda}$$

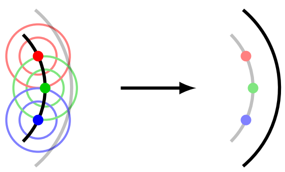
where $f = 1/T$ is the frequency of the wave, that is cycles per second (s^{-1}) or Hertz (Hz). The **amplitude** A is the magnitude of the disturbance produced by a wave.

Electromagnetic and many mechanical waves are described by sine or cosine functions such as $A\cos(2\pi(x/\lambda - ft))$, known as sinusoidal (or harmonic) waves.



Diffraction

Diffraction is the spreading out of a wave that occurs when a wave passes through a gap or around an object. Diffraction patterns may be understood using **Huygens principle**: each point on a wavefront acts a source of spherical wavelets that *interfere* i.e. add constructively or destructively (producing larger or smaller net displacement, respectively).

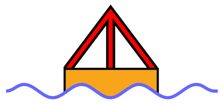


Huygens: the wavefront at a later time is found where the crests from each source point coincide

Example 5.1. Sound is an example of a longitudinal wave where particles in a solid, liquid or gas are displaced forward and backwards along the direction of travel of the wave. The pattern of density (or pressure) changes this causes in the medium are called *compressions* (high density) and *rarefactions* (low density).

For periodic waves in a solid, the particles oscillate (or vibrate) around their equilibrium positions. More generally, these oscillations are superposed on any other motion e.g. diffusion in a liquid or gas.

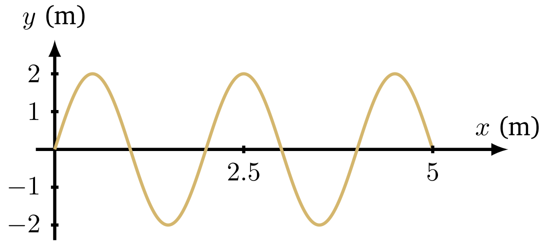
In addition to electromagnetic waves, transverse waves may be created on strings, in solids (shear waves) and bodies of water.



Exercise 5.2. A sailor observes a buoy in the sea reaching a maximum height 30 times in 2 minutes. Calculate the frequency of the wave motion. If waves in that area of the sea travel with speed 2 ms^{-1}, what is their wavelength?
[hint: use $f = N/t$, N the number of oscillations in time t]

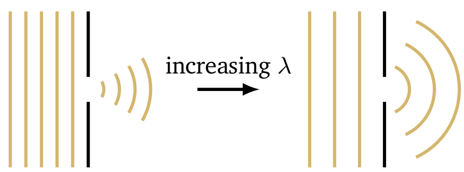
Example 5.3. Since λ is the distance travelled by a wave in one period T, we see that the relationship $v = \lambda/T$ is just the familiar speed = distance/time.

Exercise 5.4. The below plot shows a wave 0.4 s after is was started (at $x = 0$). Find the i. amplitude ii. wavelength iii. frequency and iv. speed of the wave.



Example 5.5. The wavespeed v of a mechanical wave is controlled by properties of the medium it propagates through. For example, waves travel faster on a tighter and lighter string, and sound travels faster in steel than air.

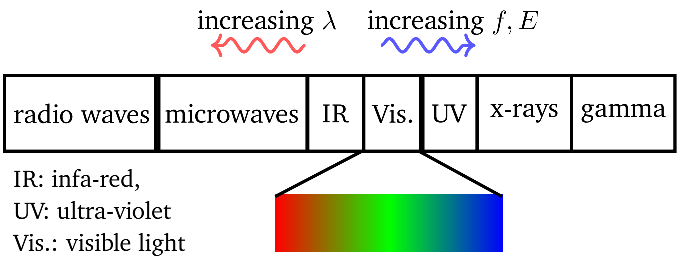
Exercise 5.6. Diffraction through a gap is appreciable when the gap with is comparable to the wavelength, and for a given gap the amount of diffraction increases with wavelength:



Produce similar sketches to show the same effect for diffraction past the edge of a barrier and explain how this means a house separated from a radio and television (TV) transmitter by a hill may have good radio signal but poor TV signal.

Electromagnetic spectrum

All electromagnetic radiation (waves) travels at the speed of light $c = 3\times10^{-8}$ ms^{-1} in vacuum and resides on a spectrum according to its wavelength or equivalently (via $c = f\lambda$) its frequency. Names e.g. radio, microwave, are used to describe bands of this spectrum which have common sources, detection methods and uses (Example 5.7).

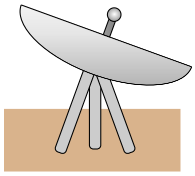


Wave	Source	Detector
radio	radio transmitter	telescope
microwave	magnetron, cell tower	aerial
infrared	photodiode	radiator
visible	lamp, TV	human eye
ultraviolet	the sun	fluorescent pigments
x-ray	particle accelerators	photographic film
gamma	radioactive nuclei	spectrometer

Example 5.7. Uses of electromagnetic radiation

Wave	Application
radio	radar, radio astronomy
microwave	cooking, telecommunications
infrared	TV remote, night vision camera
visible	photography, fibre optics
ultraviolet	forensics, sterilisation
x-ray	radiography, crystallography
gamma	medical tracers, irradiation

Exercise 5.8. Using a ground-base telescope an astronomer detects radio waves from a distant star of frequency 200 MHz.



i. Calculate the wavelength of the radio waves
ii. If the star is 20 light years away how long did the waves take to reach Earth?
iii. Suggest why the astronomer cannot detect x-rays from the same star.

Example 5.9. Radio waves are radiated when charged particles are accelerated. In a radio transmitter, a time-varying current is applied to a metal conductor called an antenna. As this current comprises oscillating electrons, radio waves are produced.

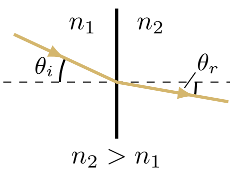
Refraction of light

Refraction is the change in speed and direction of a wave as it travels from one medium to another.

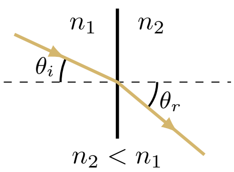
For electromagnetic radiation (light) the change in speed is described by the **refractive index** of the material n such that light travels as $c' = c/n$ in the material ($n = 1$ in a vacuum otherwise $n > 1$).

Ray diagrams are used to depict refraction events with a ray indicating the direction of propagation of an incoming or outgoing wave. The angle the incoming ray makes with the **normal** of a boundary is the angle of *incidence* θ_i and that that of the outgoing ray the angle of *refraction* θ_r.

If waves arrive perpendicular to the boundary then there is no change in direction and $\theta_r = \theta_i = 0$. Otherwise if passing from a *less to more dense* medium the change in direction is *toward* the normal ($\theta_r < \theta_i$) and *away* from the normal ($\theta_r > \theta_i$) if passing from a *more to less dense* medium.



$n_2 > n_1$
$\theta_r < \theta_i$

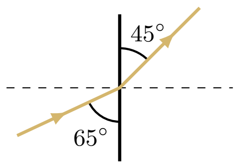


$n_2 < n_1$
$\theta_r > \theta_i$

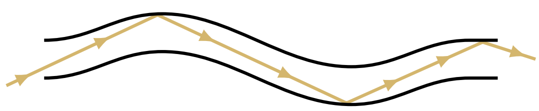
Example 5.10. For water, $n = 1.333$ and $c' = 2.25\times10^{-8}$ ms^{-1}. On the other hand, in air $n = 1.0003$ and for most intents and purposes the speed of light is unchanged.

Note in many materials n varies with wavelength λ. This is the cause of dispersion when e.g. white light is shone on a prism.

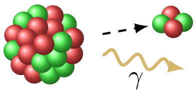
Exercise 5.11. Determine the angle of incidence θ_i and refraction θ_r below. Is region 2 more or less (optically) dense than region 1 and what does this mean in terms of their refractive indices?



Example 5.12. When $n_2 < n_1$ e.g. glass-to-air (more to less dense), at a critical angle of incidence $\theta_i = \theta_c$, θ_r reaches 90° and *no* light escapes the material. This phenomenon is known as **total internal reflection** and is the principle of operation of optical fibres where light is confined to a narrow core of doped Silica.



Nuclear radiation



Atoms of the same element have the same number of *protons* (atomic number Z). Atoms of the same element with different number of *neutrons* N are **isotopes** and may be stable or unstable.

Nuclear decay

Unstable isotopes tend to undergo **nuclear decay** to a more stable (energetically favourable) state, either

1. **Alpha decay**. A high energy Helium-2 nucleus (2 protons and 2 neutrons) is emitted,

$$^{A}_{Z}X \rightarrow \, ^{A-4}_{Z-2}Y + \, ^{4}_{2}\mathrm{He}^{2+}$$

here $A = Z + N$ is the total number of nucleons and and the emitted He nucleus is identified as an alpha particle, α^{2+}.

2. **Beta decay**. A neutron transforms into a proton, emitting an electron (beta-minus particle) in the process

$$^{A}_{Z}X \rightarrow \, _{Z+1}{}^{A}Y + \beta^{-}$$

In addition, during either process high energy gamma rays γ may be emitted as the nucleus attains a more stable state.

Activity and half life

The **activity** of a radioactive source is the number of decays per unit time (Becquerels, Bq), $A = N/t$, which depends on the type (isotope) and size (number of nuclei) of the source. The **half life** $\tau_{1/2}$ is the time taken for the activity to half. Equivalently, it is the time for half the number of unstable nuclei currently present to have decayed.

Radioactive decay is an inherently random process: we can say how long on *average* it will take atoms of a given isotope to decay, but not when a particular decay will take place.

Ionisation

Ionisation is the removal (or addition) of an electron from an atom to form a positive (or negative) ion. Each of α, β and γ radiation have the potential to cause ionisation: α is the *most* strongly ionising but the least penetrating, γ is the least ionising and most penetrating, and β is typically somewhere in-between the two extremes (Exercise 5.15).

Radiation and living matter

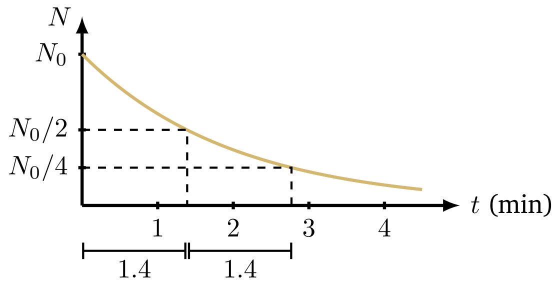
Exposure to ionising radiation can cause damage to living tissues and organs. The amount of exposure is quantified by the **absorbed dose** D, which is the energy E deposited by the ionising radiation per unit mass (Jkg^{-1} or Gy), or the **equivalent dose** H, which is the absorbed dose weighted according to the type:

$$D = \frac{E}{m} \qquad H = w_r D$$

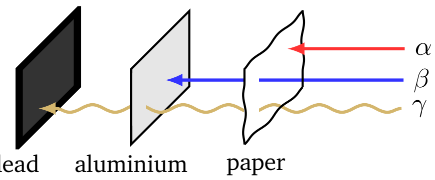
H is commonly measured in Sieverts, Sv.

Exercise 5.13. The activity of a source falls from 32 MBq to 8 MBq in 9 days. Calculate its half life. How long would it take for the activity to decrease from 8 MBq to the levels typically found in a banana, 80 Bq?

Example 5.14. Half life may be determined graphically from a plot of number of nuclei N or activity A against t. Below we find $N \to N/2$ in approximately $\tau_{1/2} = 1.4$ min.



It is a good idea to obtain $\tau_{1/2}$ from the average of several readings to reduce error.



Exercise 5.15. Match the radiation type to ionisation power and typical penetration depth.

type	ionisation power	penetration air	penetration water (tissue)
alpha	moderately	$\gtrsim 100$ m	~ 10 cm
beta	weakly	~ 10 m	$\sim 10\ \mu$m
gamma	strongly	~ 3 cm	~ 10 mm

Example 5.16. Weight factors for common radiation sources.

radiation	γ	β	α	fast neutrons
w_r	1	1	20	10

Exercise 5.17. A 0.5 kg part of the body absorbs 0.1 J of gamma radiation. Calculate the absorbed dose in Gy (Jkg^{-1}). What would be the implication of alpha instead of gamma being absorbed?

Example 5.18. Since the health implications of ionising radiation depend on the time t over which a dose is absorbed, it is often useful to calculate the equivalent dose *rate*,

$$\dot{H} = \frac{H}{t}$$

Exercise 5.19. Over a period of 30 minutes a radiation worker absorbs 20 mGy from β sources and 20 μGy from fast neutrons. Calculate the equivalent dose rate per hour (mSvhr^{-1}).

Nuclear radiation (continued)

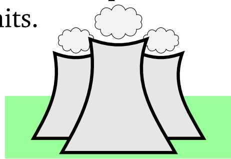
Absorbed dose then takes into account that the potential harm to biological tissue depends not only on the total amount of energy absorbed, but also the type of radiation. Example weight factors are given in Example 5.16.

Brief exposure to high equivalent doses may produce burns or radiation sickness, whilst prolonged exposure to even low doses can lead to cancer. It is thus essential to ensure levels of exposure to members of the public and in particular radiation workers remain within safe limits.

Example 5.20. The annual effective* dose limit for the general public is 1 mSv, and for a radiation worker is 20 mSv.

*The effective dose is the equivalent dose multiplied by a second set of factors w_{r2} according to the type of *tissue* that absorbed the radiation (e.g. bone $w_{r2} = 0.01$, lung $w_{r2} = 0.1$).

Exercise 5.21. The average annual equivalent dose from background radiation in the UK is around $2 - 3$ mSv. Research and list examples of natural and artificial sources that contribute to this dose.



Nuclear energy

The large amounts of energy released in nuclear reactions (e.g. versus chemical reactions) can be harnessed for energy generation. Nuclear power stations use **fission** reactions although there is continued research and development into harnessing **fusion** reactions too.

Nuclear fission

In the nuclear **fission** reactions powering nuclear power plants a heavy ^{235}U (Uranium-235) nucleus absorbs a free neutron to form an excited ^{236}U which rapidly decays into two lighter nuclei (e.g. ^{92}Kr $+^{141}$Ba) and several neutrons. These neutrons can then go to induce fission in other ^{235}U nuclei, leading to a *chain reaction*. The energy released in this process as both γ-rays and the kinetic energy of the free neutrons provides vast amounts of heat which can be converted into electrical energy.

Example 5.22. In nuclear **fusion** one instead has two small nuclei, typically isotopes of Hydrogen such as deuterium ^{2}H and tritium ^{3}H combining to form a larger ^{4}He nucleus and a neutron.

Whilst this reaction releases vast amounts of energy in the form of the kinetic energy of the products, the energy required to bring the positively charged nuclei close enough together (against their mutual electrostatic repulsion) to initiate the reaction makes it extremely difficult to achieve a self-sustaining reaction process with net energy release. For example, the temperatures required for fusion to occur necessitate the reactants are in a plasma state, which must be carefully confined e.g. using a tokamak.

SI prefixes and precision

As well as common SI units (m, kg, s, J and K) you should be aware of prefixes indicating powers of 10:

prefix	n	μ	m	k	M	G
name	nano	micro	milli	kilo	mega	giga
factor	$\times 10^{-9}$	$\times 10^{-6}$	$\times 10^{-3}$	$\times 10^{3}$	$\times 10^{6}$	$\times 10^{9}$

Alternatively, a factor of 10 may be used without a prefix, known as **scientific notation** (Exercise 5.25).

Significant figures of a numerical result are digits understood to be reliable (ignore leading zeros). Final answers should be quoted to a sensible number of significant figures (s.f.), no more than the value with the *least* number of significant figures used in the calculation.

Exercise 5.23. Write the following with a sensible prefix and unit, to the number of significant figures (s.f.).

i. $380, 120$ ms^{-1}, 2 s.f.
ii. 5.6782×10^{-9} m, 3 s.f.
iii. 0.0019285 K, 1 s.f.

Example 5.24. If the time for light to travel an unknown distance x is measured to be 0.380 ns and the speed of light is $c = 2.99792 \times 10^8$ (6 s.f.), then $x = c \cdot 0.380 \times 10^{-9} = 2.99792 \times 10^8 \cdot 0.380 \times 10^{-9} = 0.114$ m or 1.14×10^{-1} m (scientific notation) to 3s.f.

Exercise 5.25. Write the following using scientific notation and SI units without prefixes, to 3s.f.

i. $E = 130$ mJ
ii. $v = 38.511$ km^{-1}
iii. 0.09999×10^{-2} K